

Depolarisation cooling of an atomic cloud

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Abstract. – We propose a cooling scheme based on depolarisation of a polarised cloud of trapped atoms. Similar to adiabatic demagnetisation, we suggest to use the coupling between the internal spin reservoir of the cloud and the external kinetic reservoir via dipolar relaxation to reduce the temperature of the cloud. By optical pumping one can cool the spin reservoir and force the cooling process. In case of a trapped gas of dipolar chromium atoms, we show that this cooling technique can be performed continuously and used to approach the critical phase space density for BEC.

Introduction. – Adiabatic demagnetisation [1, 2] is a well established and very efficient cooling scheme which enables researchers in solid state physics to cool their samples by several orders of magnitude in a single cooling step [3, 4]. However, depolarisation processes have not yet led to a cooling concept in atomic physics. Instead, evaporative cooling which can be observed in many fields of physics is typically applied to obtain temperatures in the nK regime. This cooling mechanism was proposed and demonstrated for magnetically trapped atoms by Hess [5]. Meanwhile, it has been studied intensively [6, 7] and could also successfully be applied to atoms [8, 9] and molecules trapped in optical dipole traps [10]. Up to now, this scheme is essential to obtain degenerate atomic or molecular quantum gases and allows nowadays to generate gases with temperatures below $T = 500$ pK [11]. By means of a controllable finite trap depth U_0 , high energetic particles carrying more than the mean energy of a trapped particle are allowed to escape from the trap. Rethermalisation of the remaining particles via elastic collisions reduces the temperature of the trapped atomic cloud and at the same time produces particles which have sufficient energy to leave the trap again. In typical experiments, this technique allows one to increase the phase space density $\rho = n_0(2\pi\hbar^2/(mk_BT))^{3/2}$ by several orders of magnitude, where n_0 and m denote the peak density and the atomic mass, respectively. The ratio $\eta_{ev} = U_0/(k_BT)$ between the trap depth and the thermal energy of the cloud is commonly referred as cutoff parameter. The higher this ratio is chosen, the more energy can be carried away by a single atom and the less atoms are lost to achieve the final temperature. However, in this case the particles need more time to rethermalise, so that trap losses become more significant and finally limit the cooling process. Thus, the efficiency χ of the cooling process is defined by the gain in phase space density per atom loss

$(-d(\ln \rho)/d(\ln N))$ which can be optimised during the evaporation using the cutoff parameter. Typical experimental values of χ range up to 4. Beside the intrinsic high loss of atoms of this cooling method, in optical traps the trapping volume has to be enlarged in order to reduce depth of the trapping potential, thus the forced evaporation cooling typically does not reach the *runaway* regime where the evaporation accelerates itself.

To reduce the temperature of an atomic sample more efficiently, we suggest to transfer kinetic energy to an internal degree of freedom (spin) while the cloud depolarises via inelastic dipolar relaxation collisions. Subsequent optical pumping connects the spin reservoir to the light field and allows to remove the energy from the trapped atomic cloud. This cooling scheme does not rely on removing atoms from the sample or changing the trapping potential. Therefore, it is expected to be much more efficient than conventional evaporative cooling.

In the following, we consider N atoms in an homogenous magnetic offset field B trapped in a power-law potential, which is independently of the internal state characterised by $U(x, y, z) = c_x x^{n_1} + c_y y^{n_2} + c_z z^{n_3}$ with $\alpha = \sum_j n_j^{-1}$ and may be realised by a far off resonant optical dipole trap. Having in mind a specific element – ^{52}Cr , which has been Bose-Einstein condensed recently [12] – we focus in this letter on atoms in a stable state without both hyperfine structure ($I = 0$) and electron orbital momentum ($L = 0$), though these are no restrictions for the cooling scheme. A finite electron spin S leads to $2S + 1$ magnetic substates $|m_S\rangle$ which are energetically separated by the Zeeman energy $\Delta E_Z = 2\mu_B B$ (see fig. 1), where μ_B is the Bohr magneton. The dipole moment can cause inelastic dipolar relaxation collisions in which the total spin quantum number of both atoms is not conserved. Energy conservation requires, that for each spin flip event to a neighbouring lower ($\Delta m_S = -1$) or higher ($\Delta m_S = 1$) energetic state the energy ΔE_Z is transferred to or detracted from the kinetic energy of the colliding atoms. Moreover, we assume for the simulation, that thermalisation between the external degrees of freedom (kinetic reservoir) via elastic collisions occurs much faster than the energy transfer to the spin reservoir via dipolar relaxation. Such we can imply thermal equilibrium of the kinetic reservoir and the total energy of the kinetic reservoir is given by $E = (3/2 + \alpha)Nk_B T$. Starting from a non-equilibrium distribution of atoms across the states $|m_S\rangle$, the sample gradually relaxes via dipolar relaxation to equilibrium occupation of the states which is given by the Boltzmann distribution. For a net relaxation rate $\dot{N}_r = \sum_{i=1}^N \dot{m}_{S,i}$ which contains spin flip events to neighbouring states, the cooling rate can then be estimated using the time derivative of the total energy of the kinetic reservoir ($\dot{E} = (3/2 + \alpha)k_B(\dot{T}N + T\dot{N})$):

$$\dot{T} = \frac{\Delta E_Z \dot{N}_r}{(3/2 + \alpha)Nk_B}, \quad (1)$$

where we have neglected atom loss. Depending on the starting condition relaxation will cool or heat the sample. The latter process limited us to obtain Bose-Einstein condensation in a cloud of chromium atoms in a magnetic trap [14].

If the atoms with temperature T_0 are polarised in the energetically lowest state, transitions to the neighbouring higher energetic substate caused by dipolar relaxation collisions cool the sample. As the sample approaches equilibrium, the relaxation rate and therefore the cooling rate tend to zero. The final reachable temperature T_{eq} for a certain magnetic field is depicted in fig. 2 and can be calculated from:

$$E = N \left(\frac{3}{2} + \alpha \right) k_B T_0 = N \left(\left(\frac{3}{2} + \alpha \right) k_B T_{eq} + \Delta E_Z \frac{\sum_{i=0}^{2S} e^{-\frac{\Delta E_Z}{k_B T_{eq}} i}}{\sum_{i=0}^{2S} e^{-\frac{\Delta E_Z}{k_B T_{eq}} i}} \right). \quad (2)$$

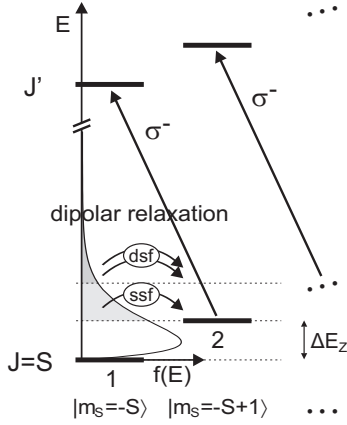


Fig. 1

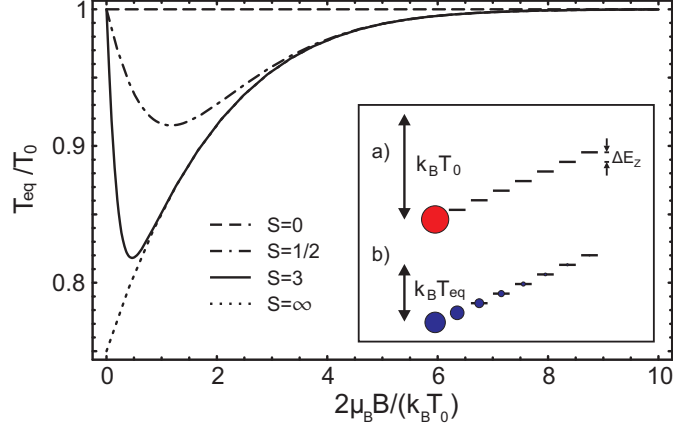


Fig. 2

Fig. 1 – Dipolar relaxation collisions into energetic higher state. Shown are the lowest two energy levels of a ground and excited state manifold of a $J \rightarrow J' = J$ -transition, respectively. The levels are separated by the Zeeman energy ΔE_Z . Also indicated is the distribution function $f(E)$ of the relative kinetic energy of a trapped polarised atom cloud in thermal equilibrium. Inelastic single (ssf) and double (dsf) spin flip transitions are only possible for atoms in the high energy tail of the distribution. Using σ^- -polarized light the sample can be polarised in the energetically lowest and dark state ($|m_S = -S\rangle$).

Fig. 2 – Equilibrium temperature of a previously polarised cloud depending on the magnetic field and the spin quantum number S . The initial (a) and equilibrium (b) situation of sample with $S=3$ is indicated in the inset.

Unlike in solids used for adiabatic demagnetisation where the phonon heat capacitance in a cryogenic surrounding ($T_0 \sim 1K$) is negligible compared to the heat capacitance of the spin reservoir, in a trapped atomic gas ($T_0 \sim 1 - 10^3 \mu K$) the heat capacitances of the spin and kinetic reservoir are of comparable magnitude. Therefore, the achievable temperature reduction is much smaller and an optimum ($T_{eq}/T_0 = (3/2 + \alpha)/(5/2 + \alpha)$) is theoretically obtained for $S \rightarrow \infty$ in the limes of $B \rightarrow 0$. However, in atomic physics the spin reservoir can be cooled very easily by optical pumping. In this way, this depolarisation process can be repeated several times or even driven in a continuous way, like we will discuss in the following. Experimentally, polarisation of the sample in the energetically lowest state $|m_S\rangle = -S$ can e.g. be accomplished using σ^- -polarised light on a $J = S \rightarrow J' = J$ transition. In this case, $|m_S\rangle = -S$ is a dark state and its population is not affected by the pumping light (see fig. 1).

In a cycle consisting of a dipolar relaxation collision and an optical pumping transition, cooling can be provided if the Zeeman energy ΔE_Z exceeds the energy ($E_{pol} \sim E_{rec} = k^2 \hbar^2 / (2m)$) needed to polarise the cloud. In principle this scheme allows to generate samples with temperatures below $T_{pol} = E_{pol} / k_B$, since in a thermal distribution high energetic atoms $E \geq \Delta E_Z > E_{pol}$ will undergo dipolar relaxation collisions and contribute to the cooling of the cloud. In this respect, dipolar relaxation can be considered as an evaporation out of the energetically lowest state, where the cutoff parameter is given by $\eta_B = \Delta E_Z / (k_B T)$. Note, the energy reduction of the sample works without thermalising collisions. The required ingredients are a high inelastic dipolar relaxation rate and a optical pumping transition back to the initial state, which is a dark state for the pumping light. A scheme based on elastic collisions (spin changing collisions) in combination with a quadratic zeeman shift of the magnetic substates

was proposed by G. Ferrari [13]. In his case, either linear or circular polarised pumping light provides the necessary pumping mechanism including a dark state.

Cooling model. – In the following, we develop a model to describe the continuously driven cooling process and study the practicability of the scheme for a sample of chromium atoms. Dipolar relaxation rates which will be used in this section have been previously studied experimentally and theoretically in our group [14]. There we found that the process is well described by dipole-dipole scattering in the first Born approximation where either no (elastic collision), one or both colliding atoms undergo a transition to a neighbouring substate ($\Delta m_S = 0, \pm 1$). If $\Delta M_S = \Delta m_{S,1} + \Delta m_{S,2} = -2, \dots, 2$ accounts for the total change in the spin quantum number of both atoms, $\Delta M_S \Delta E_Z$ is the energy which is released ($\text{sign}(\Delta M_S) < 0$) or required ($\text{sign}(\Delta M_S) > 0$) during such a collision. The inelastic dipolar relaxation rate for an atom cloud polarised in a extremal magnetic substate into the neighbouring state is given by $\dot{N}_r = \dot{N}_{dip} = -\beta N^2 / \bar{V}$, where $\beta = \langle (\sigma_1 + 2\sigma_2) v_{rel} \rangle_{therm}$ is the thermally averaged rate constant containing single and double spin-flip transitions and \bar{V} is the mean trapping volume. Hereby we introduced the collision cross sections for single spin-flip ($\sigma_1 = \xi S^3 (1 + h(k_{f,1}/k_i)) k_{f,1}/k_i$) and double spin-flip ($\sigma_2 = \xi S^2 (1 + h(k_{f,2}/k_i)) k_{f,2}/k_i$) events with $\xi = (\mu_0 (2\mu_B)^2 m)^2 / (30\pi \hbar^4)$ and the relative velocity v_{rel} , respectively. $h(x)$ includes the symmetries of the particles and is defined in [14]. Finally, the factor $k_{f,\Delta M_S}/k_i$ accounts for the different density of final states in the inelastic process and therefore ensures that the energy conservation is fulfilled:

$$\frac{k_{f,\Delta M_S}}{k_i} = \begin{cases} \sqrt{1 - \frac{m \Delta M_S \Delta E_Z}{\hbar^2 k_i^2}} & \text{if } 1 > \frac{m \Delta M_S \Delta E_Z}{\hbar^2 k_i^2}, \\ 0 & \text{otherwise.} \end{cases} \quad (3)$$

Thus, the energy transferred to spin reservoir via dipolar relaxation collisions is given by $\dot{E}_{dip} = \text{sign}(\Delta M_S) \Delta E_Z \dot{N}_{dip}$. While ΔE_Z linearly increases with the magnetic field, \dot{N}_{dip} decreases for atoms polarised in the energetic lowest substate ($|m_S\rangle = -S$). Therefore, the amount of transferred energy can be optimised by the cut-off parameter η_B . In the case of trapped chromium atoms ($S=3$), it is maximum for $\eta_{B,opt} \approx 1.31$. If the temperature of the cloud exceeds the recoil temperature ($T \gg T_{rec}$), other heating mechanisms can be neglected and we obtain a rough estimation ⁽¹⁾ for the expect cooling rate:

$$\dot{T}(\eta_B) \approx -\frac{2}{3/2 + \alpha} \sqrt{\frac{k_B}{\pi m}} \xi S^2 \{ (1 + \eta_B) S + (2 + 4\eta_B) e^{-\eta_B} \} \eta_B e^{-\eta_B} \frac{N}{\bar{V}} T^{3/2}. \quad (4)$$

Since the rate intrinsically depends on the dipolar relaxation rate, the cooling process works especially well for dense samples of atoms or molecules which possess high spin quantum numbers. The sum within the curly brackets contains both spin flip transitions whereas the contribution of the double spin flip transition is suppressed by a factor of $e^{-\eta_B}$. Moreover, we find that the time evolution of the temperature is closely connected via the mean volume ($\bar{V} \propto T^\alpha$) to the form of the trapping potential since a reduction in temperature simultaneously leads to an increase in density. While a three dimensional harmonic potential ($\bar{V} = (\sqrt{4\pi k_B T} / (\bar{\omega} \sqrt{m}))^3$, where $\bar{\omega}$ is the mean trapping frequency) results in a linear change of the temperature with the volume, a potential with $\alpha > 3/2$ – like it can be realised in a *dimple* trap – enhances the cooling process in time and yield a *runaway* behaviour.

In order to obtain a more realistic model, we include atom loss and the optical pumping process. Herby we will restrict our following considerations to a three dimensional harmonic

⁽¹⁾To calculate $\eta_{B,opt}$ and eq. (4) we approximated $(1 + h(k_{f,2}/k_i)) k_{f,\Delta M_S}/k_i = 1/2 \cdot \Theta(v_{rel} - \sqrt{2\mu_B \Delta M_S / m})$ for atoms in the lowest state.

trapping potential. Atom loss processes caused by background gas collisions and three-body recombination are characterised by the rate constants $1/\tau_{bg}$ and L_{3b} , respectively. We do not expect two-body losses, if a sufficient deep potential is provided ($U_0 \gg \Delta E_Z$ and $\dot{N}_{ev} \ll \dot{N}_{dip}$, where \dot{N}_{ev} is the evaporation rate out of the trap). The evolution of the total atom number in the trap is then described by:

$$\dot{N} = \dot{N}_{bg} + \dot{N}_{3b} = -\frac{1}{\tau_{bg}}N - \frac{L_{3b}}{V^2}N^3. \quad (5)$$

Depending on the loss process the kinetic energy of the cloud is changed on an average by:

$$\dot{E}_{loss} = 3k_B T \dot{N}_{bg} + 2k_B T \dot{N}_{3b}. \quad (6)$$

Since atoms lost in a three-body collision carry less than the mean energy of a trapped gas, the remaining cloud heats up. To account for polarisation effects, we consider the two energetic lowest states ($|m_S = -S\rangle$, $|m_S = -S + 1\rangle$) and assume a negligible population in the other states if the cloud is optically pumped. In the following, indices 1 and 2 will indicate these states, respectively. The net exchange of atoms due to dipolar relaxation is given by:

$$\dot{N}_r = \dot{N}_{dip,1 \rightarrow 2} - \dot{N}_{dip,2 \rightarrow 1} \quad (7)$$

where $\dot{N}_{dip,1 \rightarrow 2}$ and $\dot{N}_{dip,2 \rightarrow 1}$ are dipolar relaxation rates containing collisions between the atoms in the same state and atoms in state 1 and 2 which result in increase or decrease of atoms in state 1, respectively [15]. Including the pumping process, the population in state 1 and 2 evolve according to the following rate equations:

$$\dot{N}_1 = \dot{N}_r + \dot{N} \frac{N_1}{N} + ((1 - \kappa)N_2 - pN_1)\Gamma_{sc}, \quad (8)$$

$$\dot{N}_2 = -\dot{N}_r + \dot{N} \frac{N_2}{N} - ((1 - \kappa)N_2 - pN_1)\Gamma_{sc}. \quad (9)$$

Hereby, we introduced the scattering rate Γ_{sc} and the Clebsch-Gordan coefficient $\sqrt{\kappa}$ for a σ^- -transition from state 2. The term $pN_1\Gamma_{sc}$ takes into account that in the experiment the light is not perfectly polarised. Absorption and emission of light are each accompanied by a momentum transfer. Since the light scattering rate is on the order of the dipolar relaxation rate which is according to our assumptions much smaller than the elastic collision rate, the sample thermalise between two scattering events and the net energy transfer of an absorption spontaneous emission cycle reads:

$$\dot{E}_{pol} = (pN_1 + N_2)E_{rec}\Gamma_{sc}. \quad (10)$$

These equations neither consider effects of quantum degeneracy nor reabsorption of scattered light. In particular the latter will limit the cooling process of an extremely dense sample. However, effects of reabsorption can be reduced in a lower dimensional trap or in the so called *festina lente* regime [16], where the trap frequencies exceed the linewidth of the pumping transition. Thus, the change in the kinetic and potential energy of the atomic cloud is given by:

$$\dot{E} = \dot{E}_{dip} + \dot{E}_{loss} + \dot{E}_{pol}. \quad (11)$$

We solved the equations (5) and (11) numerically and optimised η_B after each time step to achieve maximum efficiency χ . The calculated time evolution of the required magnetic

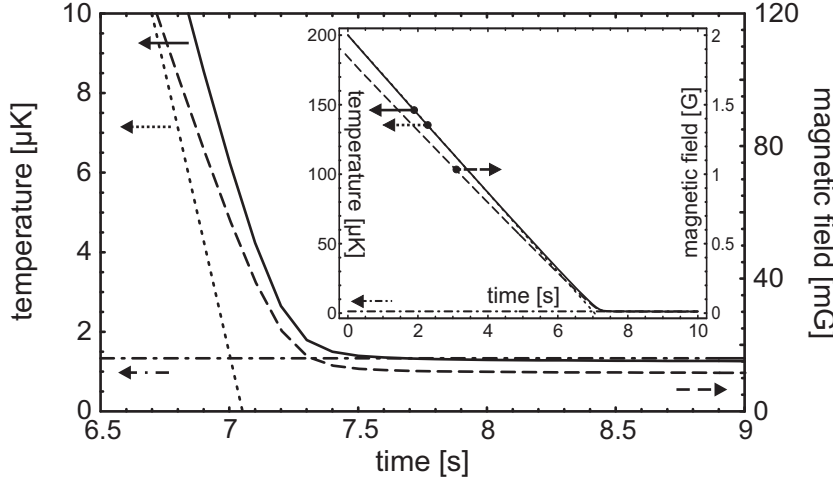


Fig. 3 – Time evolution of temperature (solid line) and magnetic field (dashed line) during the simulated cooling process. Additionally indicated are the solution of eq. 4 (dotted line) and the temperature (dashed-dotted line) corresponding to the energy needed to pump atoms from state 2 into state 1 by σ^- -polarised light. The same data are shown in the inset on a different time scale.

field and the temperature of a sample of $5 \cdot 10^6$ chromium atoms with an initial temperature of $200 \mu\text{K}$ in a harmonic trapping potential ($\bar{\omega} = 2\pi \cdot 500 \text{ Hz}$) are depicted in fig. 3. Hereby, we assumed $p = 10^{-3}$ and altered the scattering rate Γ_{sc} between $30 - 2000 \text{ s}^{-1}$ to fix the population in state 2 to 2% of state 1 during the simulation. Since additional heating caused

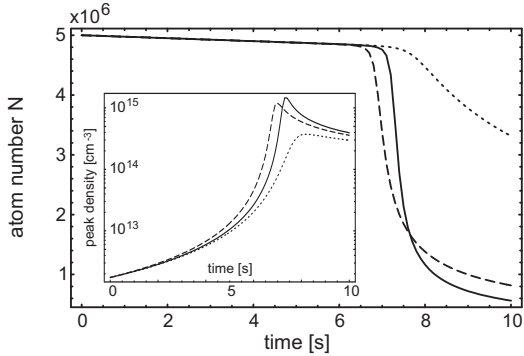


Fig. 4

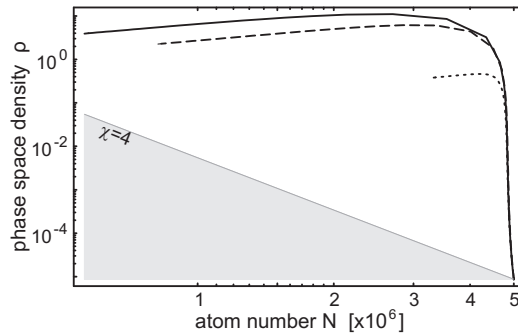


Fig. 5

Fig. 4 – Atom number and density (inset) dynamics during the cooling process. The solid line correspond to the curves shown in fig. 3 ($p = 10^{-3}$, $N_2/N_1 = 0.02$). The dotted line and the dashed curves represent simulations for $p = 10^{-2}$ which accounts for the polarisation of the pumping beam and $N_2/N_1 = 0.005$, respectively, while keeping others parameters fixed.

Fig. 5 – Phase space density gain per atom loss. The displayed curves correspond to the curves illustrated in fig. 4. The efficiency χ given by the slope of the curve, reaches a maximum value of 570 for the solid curve. Also indicated is the area (gray) where typical evaporation cooling trajectories would be expected.

by atom loss and optical pumping are negligible during the first 7 s, both the magnetic field (dashed line) and the temperature (solid line) reduce linearly in time (inset fig. 3). This process is well approximated by eq. (4) which is indicated by a dotted line. During this time, atom loss is mainly given by background gas collisions ($\tau_{bg} = 200$ s) (see fig. 4, solid line). The linear decrease in temperature leads to diverging increase in both peak density ($\propto T^{-3/2}$) (see inset fig. 3 (solid line)) and phase space density ($\propto T^{-3}$), until three-body recombination finally limits the achievable density of the atomic cloud. Preliminary experimental measurements on a trapped cloud of Cr atoms suggest a rate constant $L_{3b} \sim 10^{-41} \text{ m}^6$, which prevents us from obtaining densities much higher than $10^{15} \text{ atoms m}^{-3}$ in the simulation. A clear signature for these collisions is the more pronounced atom loss which can be observed after 7 s.

The density still rises for a little while, until heating processes have the same magnitude than the cooling processes. The cooling gets inefficient and a final almost constant temperature close to the recoil limit is reached at a magnetic field of about 10 mG. In our specific case the final temperature is below the temperature which corresponds to the mean pumping energy ($E_{pot} = \sum_{j=0}^{\infty} \kappa^j E_{rec}$) and which is indicated by a straight line in fig. 3.

In fig. 5 we illustrate the cooling process in a double logarithmic phase space density - atom number plot (solid line). The slope represents the efficiency of the cooling process as defined before. Until three-body collisions and heating caused by optical pumping limit the process, 5 orders of magnitude in the phase space density could be gained while only 5 percent of the atoms are lost. Here we obtain values of χ starting from 90 to about 570. Then the efficiency drops and finally the phase space density decreases again.

Fig. 4 and fig. 5 contain additional curves which result from simulations in which we changed either the degree of polarisation of the pumping beam $p = 10^{-2}$ (dotted line) or the ratio $N_2/N_1 = 0.005$ (dashed line). In the first case the pumping process is worsened and additional photons have to be scattered to obtain the ratio $N_2/N_1 = 0.02$. The extra heat reduces the cooling rate, so that density increases slower and atom loss is less significant. The final reachable phase space density is reduced by 1.5 orders of magnitude. Due to the improved polarisation of the sample, the cooling rate is increased in the second case and the density rises faster in the beginning of the process. However, since more photons are required to maintain the degree of polarisation of the sample the final temperature ($\sim 1.7 \mu\text{K}$) is higher and the corresponding density is lower.

Conclusion. – We studied a cooling scheme which is based on depolarisation of a polarised cloud via dipolar relaxation collisions. Our results imply that for a cloud of optically trapped chromium atoms with experimentally reachable starting conditions and parameters, the cooling scheme works extremely efficient and may allow to generate a degenerate sample. Even if this goal can not be achieved, excellent starting condition for evaporative cooling are obtained. The scheme may also be applied to other atomic (e.g. He, Cs) or molecular species which exhibit large inelastic dipolar relaxation collision rates between magnetic substates and which can be polarised optically. Especially for molecular samples it might be a way to increase the density by several orders of magnitude. The scheme allows to remove energy from a trapped sample and to increase its density without the need of thermalising collisions. The scheme may also be interesting for sympathetic cooling of mixtures of species (e.g. Cr-Rb [17]). Interspecies collisions may transfer energy and spin, so that optical pumping has not to be applied to the species which undergoes dipolar relaxation.

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